# Negative Hypothesis of Equivalence between Dynamic Magnetic Loss and Eddy Current Loss in Ferrite Grains

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The *B-H* loop of ferrites consists of two areas: one is the DC hysteresis loop and the other corresponds to the dynamic magnetic loss. The former is temperature dependent whereas the latter is temperature independent. The difference in the temperature dependence of these two areas suggests that the physical mechanism for the dynamic magnetic loss is different than that of the DC hysteresis loss. The eddy current loss in ferrite grains is a candidate for the dynamic magnetic loss. The conductivity of the ferrite grain was estimated from the experimental results of the dynamic magnetic loss. Based on the results, it was found that the conductivity is too large for iron oxide. This fact leads to some important suggestions.

Key words: ferrite, grain, eddy current loss, iron loss, dynamic magnetic loss, temperature

#### 1. Introduction

The DC hysteresis loss is interpreted as the energy loss caused by the damping torque of the magnetization vector in a magnetic material. The damping torque is phenomenally expressed in the second term of the Landau-Lifshitz-Gilbert (LLG) equation <sup>1)</sup>. The DC hysteresis loss, called simply the hysteresis loss in this paper because it occurs with the time derivative of the magnetic induction, dB/dt, generally depends on temperature, and ferrites also have the same property <sup>2)</sup>. However, it has been experimentally demonstrated that the dynamic magnetic loss in ferrites is independent of temperature <sup>3)</sup>. Accordingly, the generation mechanism of the dynamic magnetic loss is presumed to be different from that of the hysteresis loss.

In this paper, it is assumed that the dynamic magnetic loss is caused by the eddy current in ferrite grains, and the conductivity of the grains is estimated to match the experimental and computational losses. This assumption is examined based on the obtained conductivity and the result of the examination gives some novel suggestions for the LLG equation and improving ferrite property.

#### 2. Dynamic magnetic loss

Exciting a ferrite core at a high frequency yields the *B-H* loop shown as a solid line in Fig. 1. The hysteresis loop is shown as a broken line in the same figure. The hysteresis loop of ferrites does not shrink at frequencies lower than certain values, such as 1 kHz. The magnetic field intensity of the *B-H* loop is divided into two components:

$$H = H_{\rm d} + H_{\rm f},\tag{1}$$

where  $H_{\rm f}$  increases with the exciting frequency, i.e., dB/dt. The average dynamic magnetic loss,  $\overline{p}_{\rm f}$  (W/m<sup>3</sup>), per period is obtained by subtracting the area of the hysteresis loop from that of the *B-H* loop, i.e.:

$$\overline{p}_{\rm f} = \oint_C H_{\rm f} \cdot \mathrm{d}B \times f \,, \tag{2}$$



**Fig. 1** *B*-*H* loop.

where f denotes the exciting frequency.

The *B*-*H* characteristics can be expressed as

$$H = \frac{1}{\mu}B + \frac{1}{\lambda_{\rm h}}\frac{\mathrm{d}B}{\mathrm{d}t} + \frac{1}{\lambda_{\rm f}}\frac{\mathrm{d}B}{\mathrm{d}t},\tag{3}$$

where  $\mu$ ,  $\lambda_{\rm h}$  and  $\lambda_{\rm f}$  are the permeability, the hysteresis and dynamic magnetic loss parameters, respectively <sup>4)</sup>. The hysteresis characteristics are expressed as

$$H_{\rm d} = \frac{1}{\mu}B + \frac{1}{\lambda_{\rm h}}\frac{{\rm d}B}{{\rm d}t},\tag{4}$$

where  $\mu$  depends on *B* for the magnetic saturation and  $\lambda_{\rm h}$  varies to equalize the right-hand side of (4) with the measured value of  $H_{\rm d}$ . The third term on the right-hand side of (3) corresponds to  $H_{\rm f}$ , which is shown in Fig. 1, and is given by

$$H_{\rm f} = \frac{1}{\lambda_{\rm f}} \frac{\mathrm{d}B}{\mathrm{d}t} \,. \tag{5}$$

 $H_{\rm f}$  increases with increasing  ${\rm d}B/{\rm d}t$  when the ferrite core is excited by rectangular waveform voltages of different amplitudes proportional to  ${\rm d}B/{\rm d}t$ . The dynamic magnetic loss parameter,  $\lambda_{\rm f}$ , is obtained as



**Fig. 2** Experimentally obtained *B*-*H* loops at dB/dt = 300, 600 and 900 (mT/µs), respectively. ( $B_{\rm m} = 200 \text{ mT}$ ).

**Table 1** Measured dynamic magnetic loss and corresponding conductivity in ferrite grain.

dB/dt (mT/µs)	$H_{\rm f}$ (A/m)	p <sub>f</sub> (MW/m <sup>3</sup> )	$\overline{p}_{ m f}$ (MW/m <sup>3</sup> )	σ (S/m)
300	16	4.8	2.6	$7.1 \times 10^{6}$
600	29	17	10	$6.3 \times 10^{6}$
900	35	32	23	$5.2  imes 10^6$

$$\lambda_{\rm f} = \frac{\mathrm{d}B}{\mathrm{d}t} / H_{\rm f} \tag{6}$$

based on the measured values of  $H_{\rm f}$  and dB/dt. The measurements are carried out with rectangular waveform voltages because  $\lambda_{\rm f}$  can be simply determined as a function of dB/dt.  $H_{\rm f}$  depends on the magnetic flux density *B*, as shown in Fig. 1, indicating that  $\lambda_{\rm f}$  is a function of *B* as well.

Multiplying  $H_{\rm f}$  by  ${\rm d}B/{\rm d}t$  gives the instantaneous dynamic magnetic loss,  $p_{\rm f}$  (W/m<sup>3</sup>).

$$p_{\rm f} = H_{\rm f} \left(\frac{\mathrm{d}B}{\mathrm{d}t}\right) = \frac{1}{\lambda_{\rm f}} \left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)^2 \tag{7}$$

The values of  $p_{\rm f}$  and  $\overline{p}_{\rm f}$  computed from the experimentally obtained *B-H* loops shown in Fig. 2 for ferrite PC40 excited by rectangular waveform voltages are listed in Table 1<sup>3)</sup>. In the experiments, we set the maximum flux density  $B_{\rm m}$  to 200 mT and dB/dt to 300, 600 and 900 mT/µs. The values of  $H_{\rm f}$  and  $p_{\rm f}$  listed in Table 1 are the instantaneous values at 200 mT.

### 3. Eddy current loss in a ferrite grain

Ferrite is composed of grains and grain boundaries. The grains are both magnetic and conductive. The grain boundaries are so highly resistive that the eddy current flowing through them is negligible. Conversely, it is





**Fig. 3** Estimation of the eddy current loss in a ferrite grain.

conceivable that eddy currents flow inside the grains, producing the Joule heat that corresponds to the dynamic magnetic loss.

To analyze the eddy current loss in a ferrite grain, a cylindrical grain model is assumed, as shown in Fig. 3(a), where *b* designates the diameter of a grain <sup>1)</sup>. *H*, *M* and *w* are the magnetic field applied by an exciting current flowing in the winding of the core, the magnetization vector in the grain and the width of the magnetic wall, respectively. *w* is assumed to be significantly smaller than *a* and *b* shown in Fig. 3(a). In the magnetic wall, the directions of the magnetization vectors change continuously, as shown in Fig. 3(b). The direction of the magnetization vector also changes dynamically with changes in *H*; as a result, the magnetic wall moves to increase the magnetization vector that is in the same direction as *H*.

When *H* increases, the eddy current, *i*, flows in the positive  $\theta$  direction in the area where r = a/2 to b/2 and increases, because of the magnetic wall movement. dH/dt occurs in the areas not only where r = a/2 to b/2, but also inside *a*; however the electric field yielded by  $\mu_0 dH/dt$  is negligibly small compared to that generated by the movement of the magnetic wall; therefore, the eddy current caused directly by  $\mu_0 dH/dt$  is ignored.

The net magnetic field intensity applied to the magnetization vectors in the magnetic wall is given by

$$H_{\rm d} = H - H_{\rm f} \,, \tag{8}$$

where  $-H_{\rm f}$  is caused by *i*.  $H_{\rm d}$  generates the torque on the magnetization vectors. The rotations of the magnetization vectors are damping and result in the hysteresis loss. The power loss expressed in (7) with respect to  $H_{\rm f}$ , corresponds to the eddy current loss in the grain.

The average eddy current loss density in a ferrite grain, p (W/m<sup>3</sup>), is calculated with the conductivity  $\sigma$  of the grain as follows<sup>1</sup>). As *H* increases, the magnetic wall moves outward with at a rate of dR/dt, as shown in Fig. 3(b). Accordingly, the electro-motive force in the area where r = a/2 to b/2 is in proportion to the areal velocity of the movement and is given by

$$2\pi r E = M 2\pi \left(\frac{a}{2}\right) \frac{\mathrm{d}R}{\mathrm{d}t},\qquad(9)$$

where *E* is the electric field intensity and  $M = |\mathbf{M}|$ . The average eddy current loss density, *p*, is given by

$$p = \frac{\int_{\frac{a}{2}}^{\frac{b}{2}} \sigma E^2 \cdot 2\pi r \, \mathrm{d}r}{\pi \left(\frac{b}{2}\right)^2} \,. \tag{10}$$

Substituting E from (9) into (10) gives

$$p = \frac{8\sigma M^2 \left(\frac{a}{2}\right)^2}{b^2} \left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 \ln \frac{b}{a}.$$
 (11)

Assuming  $\mu_0 H \ll M$ , the average dB/dt in the grain model shown in Fig. 3(a) is expressed with respect to the electro-motive force of (9) as

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{M \, 2 \, \pi \, \frac{a}{2} \, \frac{\mathrm{d}R}{\mathrm{d}t}}{\pi \left(\frac{b}{2}\right)^2} = \frac{8 \, M \, \frac{a}{2}}{b^2} \, \frac{\mathrm{d}R}{\mathrm{d}t},\tag{12}$$

which yields p as

$$p = \frac{\sigma b^2}{8} \left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)^2 \ln \frac{b}{a},\tag{13}$$

where a depends on B as shown in the following.

Assuming b = 10 (µm) for a ferrite grain,  $\sigma$  can be estimated from (13) and the experimental data as follows. The saturation magnetic flux density of ferrite PC40 at room temperature is 500 mT<sup>2</sup>; therefore, 200 mT corresponds to  $a = \sqrt{0.3} b \cong 5.5$  (µm) which is obtained from

200 (mT) = 2×500 (mT) × 
$$\left\{ \frac{1}{2} - \frac{\pi \left(\frac{a}{2}\right)^2}{\pi \left(\frac{b}{2}\right)^2} \right\}$$
. (14)

Substituting the values of dB/dt, *a* and *b* into (13) gives the  $\sigma$  values listed in Table 1.

Considering that the conductivity of iron is

**Table 2**Measured dynamic magnetic loss andcorresponding conductivity in Ni-Zn ferrite grain.

dB/dt (mT/µs)	$H_{\rm f}$ (A/m)	p <sub>f</sub> (MW/m <sup>3</sup> )	$\overline{p}_{ m f}$ (MW/m <sup>3</sup> )	σ (S/m)
300	38	12	5.1	$15 \times 10^{6}$
400	56	22	9.0	$16 \times 10^{6}$
500	68	34	14	$16 \times 10^{6}$

approximately  $10 \times 10^6$  S/m, the values of  $\sigma$  listed in Table 1 are too large to correspond to the conductivity of ferrite grains, i.e., iron oxide. The conductivity of a ferrite grain of PC40 was estimated to be 10 S/m by an experiment where a high frequency of more than 10 MHz was applied directly to a ferrite core to short grain boundaries.

For a Ni-Zn core whose saturation magnetic flux density was 400 mT<sup>3)</sup>, similar estimation to Table 1 was carried out and its results were listed in Table 2 where  $B_{\rm m}$  was 200 mT. It is shown that the values of  $\sigma$  listed in Table 2 are too large as well.

# 4. Conclusions

The physical mechanism of the dynamic magnetic loss is assumed to be different from that of the hysteresis loss because the former is temperature independent while the latter is temperature dependent. The estimation of the eddy current loss in a ferrite grain was carried out, and a trial to prove that the dynamic magnetic loss corresponds with the eddy current loss failed. This failure suggests the followings:

- (1) The dynamic magnetic loss is assumed to be caused by the damping torque on the magnetization vector, similar to the hysteresis loss. However the temperature dependence of the two losses are different, indicating that the damping torque phenomenon for the dynamic magnetic loss should be expressed by a different term in the LLG equation, such as an additional third term that is independent of temperature.
- (2) The eddy current losses in grains of actually manufactured ferrites are small enough to allow the grains to enlarge, thus improving the magnetic properties of ferrites.

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