Equivalent circuit for Eddy Current Field in Cauer Form

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Recently, an exact and efficient modeling method for the eddy current field is found.[1)] This method expands the eddy current field to an equivalent circuit called Cauer Ladder Network (CLN). The procedure for obtaining this network and the benefits of this method are introduced here.

Consider a magnetic sheet shown in Fig. 1, where d denotes the width of the sheet, μ and σ denote the magnetic permeability and electric conductivity of the material respectively. It is supposed that exciting field $\dot{H}_0$ is applied externally. The equation for the eddy current field is given by (1) as an one-dimensional problem.

$$\frac{\partial^2 \dot{H}(x)}{\partial x^2} - j\omega\sigma\mu\dot{H}(x) = 0 \quad . \tag{1}$$

Solving this equation under the boundary condition $\dot{H}(d/2) = \dot{H}_0$ gives the magnetic field (2) and the equivalent magnetic permeability as (3).

$$\dot{H}(x) = \frac{\cos(kx)}{\cos(kd/2)}\dot{H}_0 \tag{2}$$

$$\dot{\mu} = \frac{\dot{\Phi}}{d\dot{H}_0} = \mu\frac{2}{kd}\tan\left(\frac{kd}{2}\right), \qquad \dot{\Phi} = \int_{-d/2}^{d/2}\mu\dot{H}(x)\,\mathrm{d}x = \frac{2\mu}{k}\tan\left(\frac{kd}{2}\right)\dot{H}_0 . \tag{3}$$

Here the complex variable k is defined by $k = \sqrt{-j\omega\sigma\mu}$ and $\dot{\Phi}$ denotes the total flux in the magnetic sheet. The trigonometric function divided by its argument can be expanded by the following two forms, a partial fraction expansion (4) and a continued fraction (5).

$$\frac{1}{z}\tan z = -2\sum_{n=1}^{\infty}\frac{1}{z^2 - \left[(2n-1)\pi/2\right]^2} \tag{4}$$

$$\frac{1}{z}\tan z = \frac{1}{1-}\,\frac{z^2}{3-}\,\frac{z^2}{5-}\,\frac{z^2}{7-}\,\frac{z^2}{9-}\cdots \tag{5}$$

By setting $R_c = 8/\sigma d^2, L_c = \mu$ and $L_{cn} = 2L_c/(n-1/2)^2\pi^2$, the effective complex permeability can be respectively expanded as

$$j\omega\dot{\mu} = \sum_{n=1}^{\infty}\frac{j\omega L_{cn}R_c}{R_c + j\omega L_{cn}} \tag{6}$$

$$j\omega\dot{\mu} = \frac{1}{1/j\omega L_c +}\,\frac{1}{3(R_c/2)+}\,\frac{1}{5/j\omega L_c +}\,\frac{1}{7(R_c/2)+\cdots} . \tag{7}$$

The former corresponds to the Fourier I expansion and the latter corresponds to the Cauer I expansion. The equivalent circuits for these expansions can be realized by the equivalent circuits shown in Fig. 2, Foster realization and Cauer realization respectively. These equivalent circuits can be employed for modeling of actual electric machines such as a reactor as shown in Fig. 3. In fact, the reactance of this reactor is expressed by $\mathrm{j}\omega L = \mathrm{j}\omega\dot{\mu}SN/l$, where S, N and l

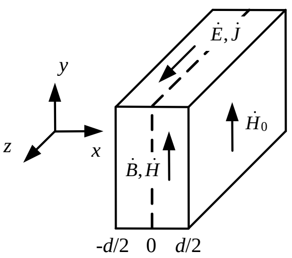


Fig. 1.　Magnetic sheet

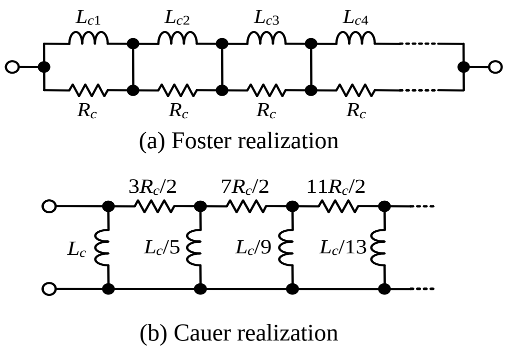


Fig. 2.　Equivalent circuits

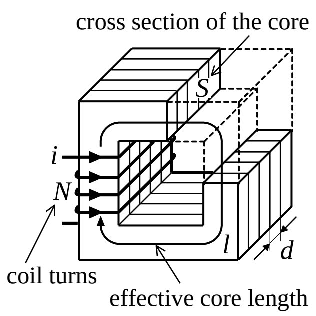
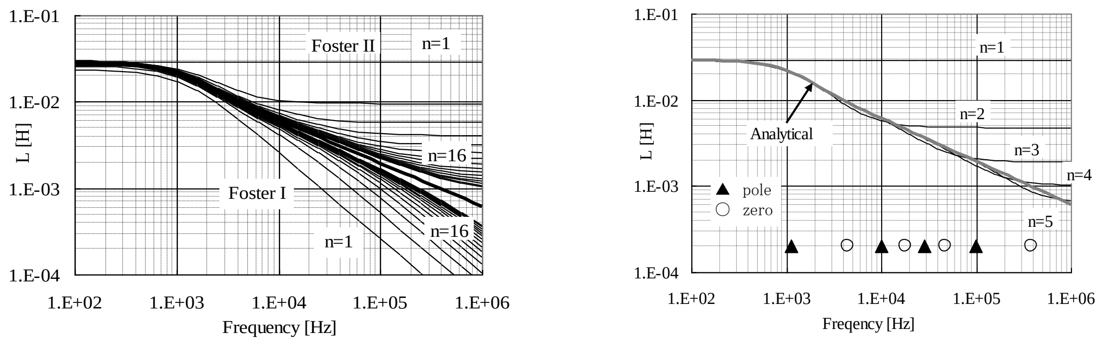


Fig. 3　Reactor



(a) Foster I and II (b) Cauer I

Fig. 4. Bode plots of Foster and Cauer expansions. ($L=\dot{\mu}$)

denote the cross section of the laminated core, the coil turns and the effective core length respectively. The examples of Bode plots with finite truncations of these networks are shown in Fig 4, where n denotes the number of the inductors. It is obvious that Cauer expansion is much effective than Foster expansion.

Recently, it was found that this Cauer realization can be expanded to arbitrary three-dimensional eddy current field as illustrated in Fig. 5.[2)] For preparation, define

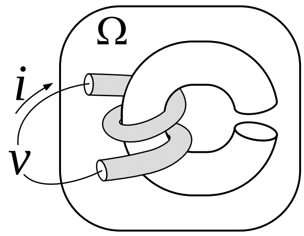


Fig. 5. Eddy current field

$$\boldsymbol{E}=\sum_{n=0}^{\infty} e_{2n}\bar{\boldsymbol{E}}_{2n}, \quad \boldsymbol{H}=\sum_{n=0}^{\infty} h_{2n+1}\bar{\boldsymbol{H}}_{2n+1} \ ,. \tag{8}$$

$$1/R_{2n}=\int_{\Omega}\sigma\bar{\boldsymbol{E}}_{2n}\cdot\bar{\boldsymbol{E}}_{2n}\mathrm{d}V, \quad L_{2n+1}=\int_{\Omega}\mu\bar{\boldsymbol{H}}_{2n+1}\cdot\bar{\boldsymbol{H}}_{2n+1}\mathrm{d}V\ . \tag{9}$$

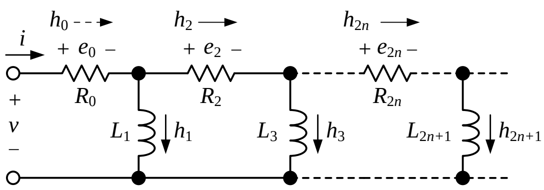


Fig. 6. Ladder network

Then the method is presented by the following steps.

***Step* 0**: Assume that the voltage v is applied externally. Solve $\nabla\times\boldsymbol{E}_0=0$ under given voltage boundary condition. Set $\bar{\boldsymbol{E}}_0=\boldsymbol{E}_0/v$ and calculate R_0 using (9). Set $\bar{\boldsymbol{H}}_{-1}=0$ and n = 1.

***Step* 1**: Solve $\nabla\times\tilde{\boldsymbol{H}}_{2n-1}=R_{2n-1}\sigma\bar{\boldsymbol{E}}_{2n-2}$ under magnetic boundary conditions. Set $\bar{\boldsymbol{H}}_{2n-1}=\tilde{\boldsymbol{H}}_{2n-1}+\bar{\boldsymbol{H}}_{2n-3}$ and calculate L_{2n-1} by using (9).

***Step* 2**: Solve $\nabla\times\tilde{\boldsymbol{E}}_{2n}=-(1/L_{2n-1})\mu\bar{\boldsymbol{H}}_{2n-1}$. Set $\bar{\boldsymbol{E}}_{2n}=\tilde{\boldsymbol{E}}_{2n}+\bar{\boldsymbol{E}}_{2n-2}$ and calculate R_{2n} by using (9).

***Step* 3**: If the finite sum of (8) converge sufficiently, then stop the calculation. Otherwise set $n=n+1$ and go to ***Step* 1**.

This method provides the network constants in Fig. 6, and simultaneously provides the spatial distribution functions $\bar{\boldsymbol{E}}_{2n}$ and $\bar{\boldsymbol{H}}_{2n+1}$. The circuit variables e_{2n} and h_{2n+1} can be obtained by real-time simulation of the ladder network. The magnetic field and the current distribution can be synthesized using (8). Furthermore, the total magnetic energy W_m and the power consumption W_R in the entire domain Ω are presented in lumped forms as

$$W_m=\frac{1}{2}\sum_{n=0}^{\infty}L_{2n+1}h_{2n+1}^2, \quad W_R=\sum_{n=0}^{\infty}R_{2n}\left(\sum_{m=n}^{\infty}h_{2n+1}\right)^2 . \tag{10}$$

In the actual electric machine designs, the nonlinearity and hysteresis property, as well as the anomaly eddy current loss, frequently become important issues. [3)] The authors hope that the proposed method can be applied to estimate the anomaly eddy current loss. However, it may not be so easy because of the moving of domain walls due to the fluctuation of the magnetic field.

Reference

1) Y. Shindo *et al.*, Simple Circuit Simulation Models for Eddy Current in Magnetic Sheets and Wires, IEEJ Trans. FMS, Vol. 134, Issue 4, pp.173-181 (2014)

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