

# Theoretical study on the finite temperature magnetism of rare earth permanent magnets

A. Sakuma<sup>1</sup>, D. Miura<sup>1</sup>, R. Sasaki<sup>1</sup>, and Y. Toga<sup>2</sup>

<sup>1</sup> Department of Applied Physics, Tohoku University, Sendai 980-8579, Japan;

<sup>2</sup> National Institute for Materials Science (NIMS), Tsukuba 305-0047, Japan)

## 1. Introduction

From an industrial viewpoint, magnetic anisotropy is the most important property of ferromagnetic materials. It governs the efficiency of magnetic media in hard disk drive (HDD) devices, permanent magnets in motors and so on. Permanent magnets have strong magnetic anisotropy above room temperature, especially in motors of hybrid and electric vehicles. They are consequently highly desired in terms of addressing energy problem.

In this symposium, we will first overview the general theory for the magnetic anisotropy at finite temperature and show how the magnetic anisotropy constants (MAC) vary as temperature increases.<sup>1)</sup> Next, we will discuss on the effects of thermal fluctuation (activation) of magnetization on the reversal (coercive) field, on the basis of magnetic viscosity. Finally, let us show some calculated results for the temperature dependence of MAC of Nd<sub>2</sub>Fe<sub>14</sub>B and mainly discuss the site dependence of the MAC's at around the room temperature.<sup>2,3)</sup>

## 2. Theoretical evaluation of the magnetic anisotropy constants at finite temperature

Suppose that the classical Hamiltonian of a magnetic system having the uniaxial anisotropy is given by

$$H = \kappa_1 \sum_i \sin^2 \theta_i + \kappa_2 \sum_i \sin^4 \theta_i + H_{ex} \quad (1)$$

where the first two terms indicate the magnetic anisotropy energy and the last term the exchange energy. Here the constants  $\kappa_1$  and  $\kappa_2$  can be expressed through the expansion coefficients of the crystal field energy in terms of the Legendre polynomials  $P_l(\cos \theta)$ , from which we have  $\kappa_1 = -3B_2^0 - 40B_4^0$  and  $\kappa_2 = 35B_4^0$  with  $B_n^0$  being the crystal field parameter. The methods to evaluate the temperature dependence of the MAC's are listed as follows:

- 1) to solve the stochastic LLG (Langevin) equation involving the thermally fluctuated field, and derive the MAC's by the magnetization curves for the applied fields parallel and perpendicular to the easy axis.
- 2) to calculate the free energy  $F(\Theta, T)$  by performing the Monte-Carlo method with the average magnetization direction of  $\Theta$ , and derive the n-th MAC's  $K_n(T)$  by fitting  $F(\Theta, T)$  with  $K_1(T) \sin^2 \Theta + K_2(T) \sin^4 \Theta$ .
- 3) to express directly the form of MAC's by means of the first order perturbation theory in terms of the anisotropy terms, which gives

$$K_1(T) = c_2 \kappa_1 + (8/7)(c_2 - c_4) \kappa_2, \quad K_2(T) = c_4 \kappa_2, \quad (2)$$

$$c_2 \equiv (1/2) \langle 3 \cos^2 \theta - 1 \rangle_{H_{ex}} = \langle P_2(\cos \theta) \rangle_{H_{ex}}. \quad (3a)$$

$$c_4 \equiv (1/8) \langle 35 \cos^4 \theta - 30 \cos^2 \theta + 3 \rangle_{H_{ex}} = \langle P_4(\cos \theta) \rangle_{H_{ex}} \quad (3b)$$

Note that  $K_1(0) = \kappa_1$  and  $K_2(0) = \kappa_2$ , since  $c_i = 1$  when  $T = 0$ .

Although the method 1) requires much computational time and resource, it is most realistic to reproduce the magnetization curves. To perform the method 2), it is useful to adopt a new technique proposed recently by Asselin et al.<sup>4)</sup> Employing this method, we have successfully

reproduced the temperature dependence of MCA's of Nd<sub>2</sub>Fe<sub>14</sub>B.<sup>3)</sup> The method 3) provides us physically transparent form of the MAC's and is the most convenient way to realize the MAC's if one further adopts the mean field approximation for the exchange term. One can understand from eqs. (2) and (3) that  $K_1(T)$  ( $T > 0$ ) is determined by  $\kappa_1$  and  $\kappa_2$ , and further by  $H_{ex}$  through  $\langle P_l(\cos \theta) \rangle_{H_{ex}}$ . Additionally, according to the Callen-Callen theory,<sup>5)</sup> one should note the relation  $\langle P_l(\cos \theta) \rangle_{H_{ex}} \approx \langle \cos \theta \rangle_{H_{ex}}^{l(l+1)/2} = \langle m \rangle_{H_{ex}}^{l(l+1)/2}$  where  $\langle m \rangle_{H_{ex}} = M(T)/M(0)$ . Recently, we have confirmed by using the methods 2) and 3) that the approximate relation  $\langle P_l(\cos \theta) \rangle_{H_{ex}} \approx \langle m \rangle_{H_{ex}}^{l(l+1)/2}$  holds in the wide range of temperature as far as  $k_B T < H_{ex}$  is satisfied. Thus, the MAC's can be expressed by using  $\langle m \rangle_{H_{ex}} = M(T)/M(0)$  as  $K_1(T) = (\kappa_1 + 8/7\kappa_2)\langle m \rangle_{H_{ex}}^3 - 8/7\kappa_2\langle m \rangle_{H_{ex}}^{10}$  and  $K_2(T) = \kappa_2\langle m \rangle_{H_{ex}}^{10}$ . In Fig. 1, we show the calculated results of the temperature dependence of MAC's of Nd<sub>2</sub>Fe<sub>14</sub>B based on the above expressions. Here, we input the experimental data of  $\langle m \rangle_{H_{ex}} = M(T)/M(0)$  and took into account for  $\sin^6 \theta_i$  term in addition to  $\kappa_1$  and  $\kappa_2$  terms. One can recognize that the above expressions can work well for the complex compounds like Nd<sub>2</sub>Fe<sub>14</sub>B.

In addition, we should emphasize here that  $K_n(T)$  ( $T > 0$ ) is dominated by  $H_{ex}$  as well as  $\kappa_1$  and  $\kappa_2$ , as mentioned above. This implies that the  $K_n(T)$  at surfaces or interfaces exhibit larger decrement with temperature than those inside the bulk. Figure 2 shows the temperature dependence of  $K_1(T)$  both for  $H_{ex} = 350$  and  $175$  in units of kelvin.<sup>1)</sup> One can see that the  $K_1(T)$  values for  $H_{ex} = 175$  [K] is much smaller than those for  $H_{ex} = 350$  [K] as bulk values, when the temperature is above 200 [K], which leads us to consider that the magnetization reversal takes place by a smaller field at the surfaces or interfaces of grains in magnets.

In the symposium, we will discuss the site dependence of the MAE and the effects of thermal activation on the reversal field in Nd-Fe-Bd.

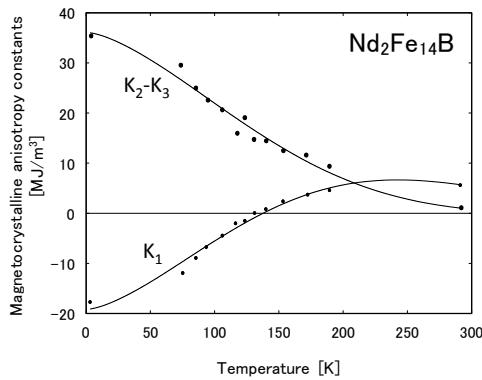


Fig.1 Temperature dependence of  $K_1$  and  $K_2 - K_3$  of Nd<sub>2</sub>Fe<sub>14</sub>B. The dots are the experimental data.<sup>6)</sup>

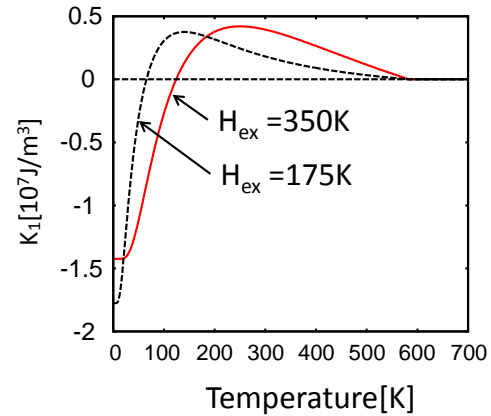


Fig. 2 Temperature dependence of  $K_1$  for the exchange energy  $H_{ex}=350$  and  $175$  [in units of K] in Nd<sub>2</sub>Fe<sub>14</sub>B.

#### References

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